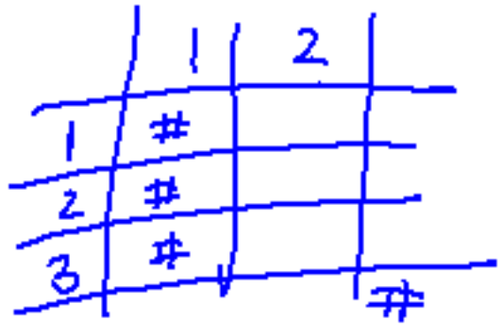
Joint Probability Distributions

- **Just like.....** 2 way tables



- **Except....**
  no numbers, ~~[redacted]~~ have probabilities in table
- **One variable is...**
  row variable (x)

- **One variable is...**
  column variable (y)

Example:

Joint Prob. Distr.

X \ Y	1	2	3	Total
1	0.32	0.14	0.19	0.65
2	0.17	0.06	0.12	0.35
Total	0.49	0.2	0.31	1

rows: 2
columns: 3
cells: 6

Let's look at X and Y separately:

For X, what is the probability distribution?

X	1	2
P(X)	0.65	0.35

For Y, what is the probability distribution?

Y	1	2	3
P(Y)	0.49	0.2	0.31

Questions:

1. P(X=1) = 0.65
2. $P(Y=2|X=1) = \frac{0.14}{0.65} = 0.2154$
3. P(Y=2 and X=1) = 0.14
4. P(Y=3) = 0.31

Now what if we have the following *independent* 2 variables:

X	1	2
P(X)	0.25	0.75

P(X=1) P(X=2)

Y	1	2	3
P(Y)	0.18	0.42	0.4

We want to combine the two together, in a joint probability distribution:

Y

X	1	2	3	Total
1	P(X=1 ∩ Y=1) 0.045	P(X=1 ∩ P(Y=2) 0.105	0.1	0.25
2	0.135	0.315	0.3	0.75
Total	0.18	0.42	0.4	1

$P(X=1 \cap Y=1) = P(X=1) \cdot P(Y=1)$

$(0.25)\ (0.18)$

Try the worksheet on the next page!

#1

a)

Y	1	2
P(Y)	0.66	0.34

b) $P(X=1 \cap Y=2) =$ 0.08

c) $P(Y=1 \mid X=3) = \frac{0.14}{0.21} =$ 0.6667

(2)

X \ Y	1	2	
1	0.08	0.12	0.2
2	0.20	0.30	0.5
3	0.12	0.18	0.3
	0.4	0.6	1

X \ Y	1	2	3	
1	0.06			0.2
2		0.1		0.4
3			0.18	0.4
	0.3	0.25	0.45	1

$$0.06 = (0.3)(P(X=1))$$

$$0.1 = P(Y=2) \cdot 0.4$$

A random variable R is normally distributed with a mean of 23 and a standard deviation of 2.45.

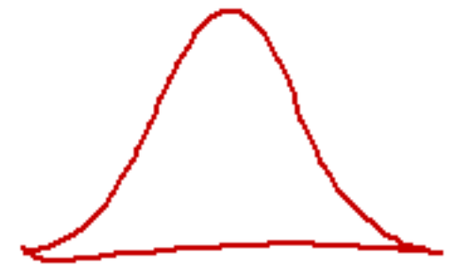
Find the following:

a) $P(N = 40)$

b) $P(N > 30)$

c) $P(22 < N < 31)$

d) $P(N < 15)$



a) 0

b) 0.002

c) 0.6579

d) 5.468×10^{-4}